



1. Let S be the set of all integers of the form $Pn + 1$ where P is a prime number greater than 5. Let N be the largest integer that divides every member of S . Find, with proof, the value of N .
2. In triangle ABC , $(\cos B)(\cos C) = \cos A$. Find, with proof, the numerical value of $(\tan B)(\tan C)$.
3. Suppose that $n + 1$ boys are lined up shoulder-to-shoulder from left to right in a straight line. Prove that it is always possible to select $n + 1$ boys to take one step forward so that, going from left to right, their heights are either consistently increasing or consistently decreasing.
4. The lengths of the sides of triangle ABC are in the ratio of 4:5:6. The bisector of the largest angle of the triangle is drawn. Prove that one of the two triangles formed also has sides whose lengths are in the ratio of 4:5:6.
5. All the factors of the polynomial $P(x) = ax^3 + bx^2 + cx + d$ are linear with integer coefficients, and neither a nor d are zero or one. Find all possible pairs (a, d) , and prove that you have found them all.

3 3 ± 3 6LQFH 3 LV D SULPH DQG 3 ! 3 LV RG
 DQG 3 PXVW EH FRQVHFXWLYH HYHQ QXPEHUV 7K
 PXOWLSOH RI3 ZKLGKYLWDEOH E\ 3\$DQB 3VLQFHUH ±
 WKUHH FRQVHFXWLYH LQWHJHUV RQH RI WKHP PXVW
 HLWKHU ± PXVW EH D PXOWLSOHGLYLVLWDEOH HEIRUH
 ,I 33 ,I 3 3 6LQFH LV WKH JFG RI DQ
 N=24 ODUJHVW

RV% FRV& %FRFRV&± V%QVLQ&

± FRV% FRV& VLQ% VLQ&

7KHUHIRUH FRV% FRV& VLQ% VLQ& VR WKDW

$$\frac{VLQ}{FRV} \quad WDI$$

\$VVXPH LW LV LPSRVVLEOH WR ILQG Q ER\ V LQ DVF
 :H ZLOO VKRZ WKDW LW LV WKHQ SRVVLEOH WR ILQG Q
 6WDUWLQJ ZLWK WKH ILUVW ER\ ZH IRUP D 'FOXE' LQ V
 ER\ WDOOHU WKDQ KLP WKH ILUVW ER\ WDOOHU WKDQ
 WKDQ Q ER\ V LQ WKLV FOXE 1H[W ZH FUHDWH D VHFRQ
 WKH ILUVW FOXE DQG FKRRVLQJ WKH ILUVW ER\ ZKR LV
 ILUVW ER\ LQ WKH VHFRQG FOXE DQG FRQLQXLQJ LQ W
 ER\ V LQ WKH VHFRQG FOXE 1RWH WKDW HDFK ER\ LQ V
 ER\ LQ WKH ILUVW FOXE ZKR LV WDOOHU WKDQ KLP RW
 1RZ IURP DPRQJ WKH ER\ ZKR DUH QRW LQ HLWKHU R
 FOXE VDUWLQJ ZLWK WKH ILUVW ER\ QRW LQ WKH ILU
 WKDQ Q ER\ V LQ WKH WKLV FOXE DQG HDFK RI WKHVH
 ZKR LV WDOOHU WKHQ KLP :H FRQLQXH LQ WKLV ZD\
 PRUH WKDQ Q PHPEHUV DQG HDFK PHPEHU RI HDFK FO
 FOXE

7KHUH DUH DWPRUW LQ WKHVH Q FOXEV VR QRZ OHW X
 DQ\ RI WKHVH FOXEV +H PXVW IROORZ D WDOOHU ER\
 WK

/HW WKH VLGHV RI WULDQJOH \$%& KDYH OHQJWKV
ORVV RI ~~SHOHLDVOWWH~~ ORQ\$%&V% VLGH RI
LV WKH ODUJHVW DQJOH% L'QVZUWKHF DQJQH ELVHFWRU
\$£DW SRLQW ' 6LQFH WKH OHQJWKV RI FRUUHVSRRGLC
VLPLODUDWHLSDQRSDRWLRQDO ZH QHHG RQO\ SURYH
WK~~BWRKUMDR~~ JOHV IRUP\$%& LV VLPLODU WR
6LQFH ERWK WULDQJOHV \$%& DQG \$' % DUH DFXWH ZH
WKDWWKLDVQJONZRDUH VLPLODU

0HWKRG

8VLQJ WKH DQJOH-ELVHFWRU ZKKFRUHP
I

I 6LQFH WKH UDWLRRI \$% WR \$' LV
\$% \$£ 7KX\$' % DQ\$%& KDYH WZR SDLUV
\$' \$%
RI SURSRUWLRQDO VLGHV DQG VKDUH WKH LQFOXGH
DQJOH \$ 7KHUHIRUH WKH WZR WULDQJOHV DUH VLF
SUR\$Y% QJOVR KDV VLGBUHZKRVH OHQJWKV
LQ WKH UDWLR RI

0HWKRG

6\$QFH DQ\$%& VKDUH DQJOH \$ ZH QHHG RQO\ ILQG
RQH DGGLWLRQDO SDLU RI FRQJUXHQW DQJOHV \$Q
FDQQRW EH FRQJUXHQW WR DQJOH \$%& , LW LV FR
WR DQJOH & WKHQ WULDQJOH %' & ZRXOG KDYH WR
7QLRQD KDSSHQ LI WKH PHDVXUH RI DQJOH \$%&
ZLFWWKDW RI DQJOH & \$
8VLQJ WKH /D\$%& & RVLQHVR RQ

FR\$% -

FR FRV-

FR&FR& - - 7KHUHIRUH P &

7KXV WULDQJOH \$%& LV VLPLD\$% DVORV WUKDDQV DGH\$' %ZKE
OHQDWLQVWKH UDWLR RI

