THE 2011–2012 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box, or more than one box, is marked. Zero points are given for an incorrect answer. Note that wild guessing is apt to lower your score. When time is up, give your answer sheet to the proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

5	8	x	4

1. In the puzzle at the right, the number in each empty square is obtained by adding the two numbers in the row directly above. For example, 5 + 8 = 13. What is the value of x?

   (A) 2 (B) 3 (C) 6 (D) 7 (E) 9

2. A circle passes through the points (0, 0), (0, 2) and (4, 0). What is the area of this circle?

   (A) 5 (B) 8 (C) 9 (D) 10 (E) 16

3. Tom found the value of 3^{21} = 10,4A0,353,20B. He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?

   (A) 0 (B) 2 (C) 3 (D) 6 (E) 8

4. In determining standings in a certain hockey league, a team receives 3 points for each win, 1 point for each tie, and –1 point for each loss. After playing 50 games, the Ducks have a total of 76 points. How many more wins than losses do the Ducks have at this time in the season?

   (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

5. Let x = m + n where m and n are positive integers satisfying $2^6 + m^n = 2^7$. The

   (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

7. If the measure of ABE is 6 degrees greater than the measure of DCE, compute the number of degrees in the measure of FD.

(A) 6 (B) 8 (C) 10 (D) 12 (E) 16

8. If q and r are the zeros of the quadratic polynomial $x^2 + 15x + 31$, find the quadratic polynomial whose zeros are $q + 1$ and $r + 1$.

(A) $x^2 + 17x + 31$ (B) $x^2 + 15x + 33$ (C) $x^2 + 13x + 17$

(D) $x^2 + 19x + 37$ (E) None of these

9. The makers of Delight Ice Cream put a coupon for a free ice cream bar in every 80th bar they make. They put a coupon for 2 free bars in every 180th bar and a coupon for 3 free bars in every 300th bar. If they put all three coupons in every nth bar, compute n.

(A) 1200 (B) 1800 (C) 2400 (D) 3600 (E) 5400

10. Starting at opposite ends of a straight moving walkway at an airport, which travels at a constant rate of k ft/sec, Don and Debbie walk towards each other (Don moving in the direction the walkway is moving, Debbie moving against the direction the walkway is moving). They meet at a point one-seventh of the way from one end of the walkway. If they were on a normal (non-moving) floor they would each walk at a rate of 3 feet per second. Determine the value of k.

(A)

20. The number 2011 can be written as $a^2 \quad b^2$ where a and b are integers. Compute the value of $a^2 \quad b^2$.

(A) 2018041 (B) 2022061 (C) 2024072 (D) 2026085 (E) 2033051

21. Consider the following system of equations: (1) ax + by = c and (2) dx + ey = f (c 0, f 0). When x = 0, equation (1) yields y = 3 and (2) yields y = 6. When y = 0, (1) yields x = -3 and (2) yields x = 3. What is the common solution (x, y) for the system?

(A) (1, 2) (B) (2, 6) (C) (4, 1) (D) (6, 2) (E) (1, 4)

22. Rectangle ABCD has sides of length 3 and 4. Rectangle PCQD is similar to rectangle ABCD, with P inside rectangle ABCD. Compute the distance from P to AB.

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) $\frac{21}{17}$ (E) $\frac{27}{25}$

23. Let S(n) = $[\sqrt{1}] + [\sqrt{2}] + \ldots + [\sqrt{n}]$ where [k] is the greatest integer less than or equal to k. Compute the largest value of k < 2011 such that S(2011) – S(k) is

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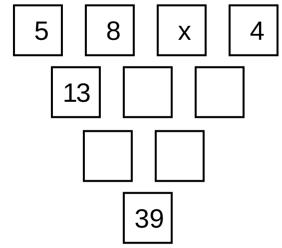
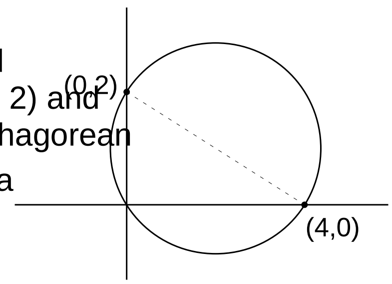
SOLUTIONS



1. A The entries in the four empty boxes, from top to bottom and left to right, are $x + 8$, $x + 4$, $x + 21$ and $2x + 12$. Then $(x + 21) + (2x + 12) = 39$ or $3x + 33 = 39$, and $x = 2$.



2. A Because the angle at (0, 0) is a right angle, it is inscribed in a semicircle, which makes the segment connecting (0, 2) and (4, 0) a diameter. Using the distance formula (or the Pythagorean Theorem), the diameter of the circle is $\sqrt{20}$, making the area $\frac{1}{4}(20\pi) = 5\pi$

3. D Of course, one could compute the value of 3^{21} directly, but that would take some time and might lead to careless errors. A more general approach is as follows. Let's find B first. The powers of 3, taken in order from 3^1, end in the repeating pattern 3, 9, 7, 1. Since 21 is one more than a multiple of 4, B = 3. Since 3^{21} is a multiple of 9, its digits must sum to a multiple of 9. Since the known digits and B have a sum of 21, the missing digit A must be 6.

4. B Let W = the number of wins, T = the number of ties, and L = the number of losses. W + T + L = 50 and 3W + T – L = 76. Subtracting the first equation from the second gives 2W - 2L = 26 and W – L = 13.

5. E If $2^6 + m^n = 2^7 = 2(2^6)$, then $m^n = 2^6$. Further,

$$2^6 \quad (2^2)^3 \quad (2^3)^2 \quad (2^6)^1$$

Thus, the possible values for m and n are

m = 2; n = 6 ⇒ x = 8
$m = 2^2 = 4$; n = 3 ⇒ x = 7
$m = 2^3 = 8$; n = 2 ⇒ x = 10
$m = 2^6 = 64$; n = 1 ⇒ x = 65

Thus, the sum of the possible values for x is 8 + 7 + 10 + 65 = 90.

6. D The numbers are 11, 22, 33, 44, 55, 66, 77, 88, 99, 12, 24, 36, 48, and 15. Trial and error will work but takes some time. However, writing a two digit integer as 10a + b and using the definition you find that only numbers with b = a, b = 2a, and b = 5a work. Therefore, there are 14 cwazy numbers between 10 and 100.

7. A Represent m∠FBC as 180 – (x + 6) = 174 – x. Also, m∠DCE = m∠BCF = x. Then m∠AFD = 180 – (174 – x) – x = 6.

8. C We could find the zeros of the given polynomial, increase each by 1, and use them to find the answer. However, that is time consuming. Here are two shorter methods.

Method 1: IC BT /TT0 1 C BT /TT0 1 T4</MCID 6 >>BDC 0 -1.15 TD (8)Tj (.)L0k0 scn Tf

13. A $\text{Log}_2 4 = 2$ and $\log_4 2 = \frac{1}{2}$. Using the triangle inequality, we have

$$\log_3 x < 2 + \frac{1}{2} \quad \text{and} \quad \log_3 x + \frac{1}{2} > 2$$

Therefore, $\log_3 x < \frac{5}{2}$ Ÿ $x < 3^{\frac{5}{2}}$ or $x < 9\sqrt{3}$ and

$$\log_3 x + \frac{1}{2} > 2 \quad \text{Ÿ} \quad \log_3 x > \frac{3}{2} \text{ or } x > 3\sqrt{3}.$$

Therefore, the set of all possible values of x is $3\sqrt{3} < x < 9\sqrt{3}$. Since $3\sqrt{3}$ | 5.2 and $9\sqrt{3}$ | 15.6, choice A (5) is the only choice that is not possible.

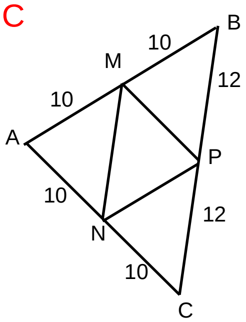
18. E Using x = 5, we obtain $2f(5) + f(-4) = 25$
Using x = –4, we obtain $2f(-4) + f(5) = 16$.
Multiplying the first equation by 2 and subtracting the equations we obtain
$-3f(5) = -34$ from which $f(5) = \frac{34}{3}$.

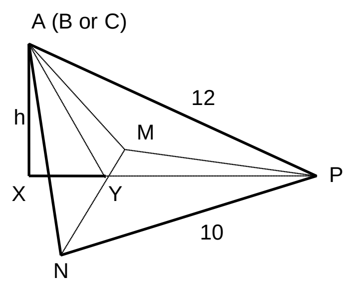
19. E Let m be the slope of the line tangent to the ellipse. The equation of the tangent

23. D Since $\sqrt{1936} = 44$ and $\sqrt{2025} = 45$, all numbers from $[\sqrt{1936}]$ to $[\sqrt{2011}]$ must equal 44. If K ≥ 1936, S(2011) − S(k) = $[\sqrt{2011}] + [\sqrt{2010}] + \ldots + [\sqrt{K\quad 1}]$ = 44(2011 − K) = (4)(11)(2011 − K). Therefore, S(2011) − S(K) will be a perfect square for 2011 − K = 11, and K = 2000.

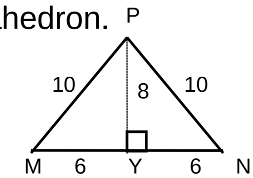
24. C &DOO WKH WZR QXPEHUV WKDW $QQH FKDQJHV GXUL
Before the operation, the sum of the squares of these two is $a^2+2a+1+a^2-2a+1 = 2a^2 + 2$, whereas after the operation the sum of the squares is simply $2a^2$. Since the other numbers do not change, we see that the sum of the squares of all nine numbers goes down by two during every operation. Because
$[(-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2 + 0^2+1^2+2^2+3^2+4^2 + 5^2 + 6^2] - (0^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2) = 182$, Anne performed 182 ÷ 2 = 91 operations

25. C

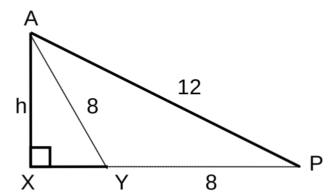




We need to find the area B of triangular base MNP (whose sides are 10, 10, and 12) and the length of h, the altitude of the tetrahedron.



It is easy to see that the area of triangle MNP is ½(8)(12) or B = 48 square units.



Next we find the length of h. In the middle diagram above, triangle PYA has sides PA = 12, PY = 8 and AY = 8. Thus, triangle PYA is obtuse, as shown. Using the Law of Cosines on triangle PYA.

$$144 = 64 + 64 - 128\cos(\angle APY) \quad \text{and} \quad \cos(\angle AYP) = \frac{1}{8}$$

Therefore, $\cos(\angle AYX) = \frac{1}{8}$, making XY = 1 and $h = 3\sqrt{7}$. Hence, $V = \frac{1}{3}Bh = \frac{1}{3}(48)(3\sqrt{7}) = 48\sqrt{7}$

The desired ordered pair is (48, 7)